Random Stuff 4

14 September 2024 14:18

Q> $ac + b + c = bc + a + 66$, $a + b + c = 32$, $a, b, c \in \mathbb{Z}^+$, $a > c$. Find a.

Ans:- $ac + b + c = bc + a + 66$

$\Rightarrow c(a-b) - (a-b) + c = 66$

$\Rightarrow (c-1)(a-b) + c - 1 = 65$

$\Rightarrow (c-1)(a-b+1) = 65 = 5 \times 13$

Case 1:- $c - 1 = 1 \Rightarrow c = 2$, $\Rightarrow a - b + 1 = 65 \Rightarrow a - b = 64$

$a + b + 2 = 32$

$\Rightarrow a + b = 30$

$\Rightarrow a = 47$

$b = -17$ → not possible

Case 2:- $c - 1 = 5 \Rightarrow c = 6$, $a - b + 1 = 13 \Rightarrow a - b = 12$

$a + b = 26$

$\Rightarrow \boxed{\begin{matrix} a = 19 \\ b = 7 \end{matrix}}$ possible

Case 3:- $c - 1 = 13$, $c = 14$, $a - b + 1 = 5 \Rightarrow a - b = 4$

$a + b = 18$

$a = 11$

$b = 7$ not possible as $c > a$

Case 4:- $c - 1 = 65$, $c = 66$, $a - b + 1 = 1 \Rightarrow a - b = 0$

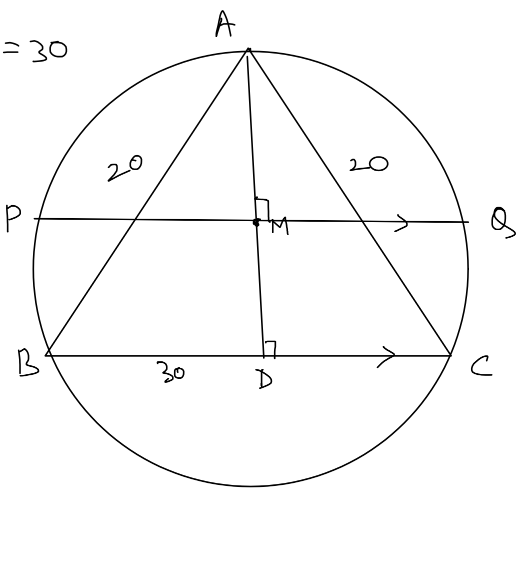
$a + b = -34$ $\Rightarrow$ not possible

Q> ΔABC is iscoceles. $AB = AC = 20$, $BC = 30$

M is midpoint of AD, $AD \perp BC$

$PQ \parallel BC$ and P, M, Q collinear.

Find the length of PQ.



Ans:- We can solve it using co-ordinate geometry.